Year 13 Further CORE Mathematics	Working towards expected outcomes Your child is not yet making the expected progress within this course.	Working at expected outcomes Your child is achieving the expected progress for this point within the course.	Working beyond expected outcomes Your child is exceeding the expected progress.
Autumn Term	Students working **towards** expected outcomes in Y13 can: • Convert between Cartesian (x, y) and polar (r, θ) coordinates using $x = r\cos\theta$, $y = r\sin\theta$ and sketch simple curves (e.g. $r = a$, $r = a\cos\theta$). • Set up the area integral $A = \frac{1}{2}\int r^2\,d\theta$ for a single petal or loop of a polar curve r = basic trig function, and evaluate it. • Obtain dy/dx for a polar curve and identify where the tangent is horizontal or vertical. • Solve homogeneous second-order DEs of the form $y'' + a y' + b y = 0$ by forming and solving the auxiliary equation and writing the complementary function.	Students working **at** expected in Y13 can: • Sketch more complex polar curves (e.g. $r = a + b\cos n\theta$, $r^2 = a^2\cos 2\theta$); determine symmetry, intercepts and loop-limits. • Find the exact area for one complete loop of $r = a\sin n\theta$ or $r = a + b\cos n\theta$ by choosing the correct θ-interval. • Derive and simplify dy/dx for r(θ) using product and chain rules; solve for θ where • $dy/d\theta = 0$ (tangent parallel to initial line) • $dx/d\theta = 0$ (tangent perpendicular to initial line). • Solve non-homogeneous DEs $y'' + a y' + b y = f(x)$ for f(x) a polynomial, exponential or trig function by undetermined coefficients, and write the full general solution.	Students working **beyond** expected in Y13 can: **In addition to the skills listed under 'Working At' for this topic, students working beyond expected outcomes can:** • Tackle boundary-value problems (e.g. $y(0) = 0$, $y(L) = 0$) for second-order DEs and discuss existence and uniqueness of solutions. • Solve cases where f(x) matches a complementary-function term by finding the correct modified particular integral. • Analyse simple harmonic motion $y'' + \omega^2 y = 0$ to derive expressions for amplitude, period and phase from given initial conditions; model vertical spring systems. • Solve damped and forced oscillations $y'' + 2\zeta\omega y' + \omega^2 y = F\cos(\Omega t)$ to find steady-state amplitude and phase shift; discuss critical, under- and over-damping.

Spring Term	• Apply one set of initial or boundary conditions to determine the constants in a general solution Students working **towards** expected outcomes in Y13 can: • Recall definitions and sketch graphs of sinh x, cosh x and tanh x • Prove basic hyperbolic identity $\cosh^2 x - \sinh^2 x = 1$ and solve simple equations sinh x = k or cosh x = k. • Differentiate sinh x, cosh x and tanh x from their exponential definitions and integrate them to recover the same functions. • Evaluate improper integrals by interpreting $\int_a^{\infty} f(x)\,dx = \lim_{R\to\infty} \int_a^R$	• Classify complementary-function solutions by the sign of the auxiliary discriminant and relate to oscillatory or exponential behaviour. Students working **at** expected in Y13 can: • Use logarithmic forms of inverse hyperbolic functions (e.g. $\sinh^{-1} x = \ln(x + \sqrt{(x^2 + 1)})$) in integration and solve more involved hyperbolic equations. • Integrate expressions involving inverse trig and partial fractions, for example $\int dx/(a^2 + x^2)$, $\int x^2/[(x+1)(x^2+1)]\,dx$. • Apply the mean-value result to real-world contexts (e.g. average velocity over a time interval). • Differentiate y = arcsin(g(x)), y = arccos(g(x)), y = arctan(g(x)) by	• Use polar area integrals to find the area between two intersecting polar curves or within multiple loops by subtracting two $\frac{1}{2} \int r^2\,d\theta$ integrals. Students working **beyond** expected in Y13 can: • Tackle hyperbolic-trig integrals via substitutions (e.g. x = sinh t for $\sqrt{(1 + x^2)}$ integrals) and compare with their trigonometric analogues. • Analyse the behaviour of inverse-trig derivatives in composite functions, including error estimates in linear approximations. • Apply the (mean-value) theorem to prove function properties (e.g. f strictly increasing ⇒ f' > 0 somewhere in each subinterval). • Use partial-fraction and inverse-trig techniques to solve first-order

	f(x) dx or by splitting at a discontinuity, for simple power-law or rational functions. • Compute the mean value of f on [a, b] via $\bar{f} = 1/(b-a) \int_a^b f(x)\,dx$.	implicit differentiation and use in growth/decay problems.	nonlinear DEs by separation of variables in unfamiliar modelling contexts (e.g. logistic growth).